Approximate Angular Directional Representation of Geometrical Objects

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Consider a scenario where a military intelligence ground-operative encounters an array of radars during a reconnaissance mission in an enemy territory, where the shape of the array is important to determine its purpose. However, he does not have requisite gadgets to make any accurate positional assessment about the array. He starts noting down the approximate angular direction of each radar from its previous radar position in the array, with respect to the North (say, by observing the Pole Star in the sky). Thus, he tours around each point in the array and records this information. Obviously, without an appropriate gadget the angular directions "estimated" will be very qualitative in nature. In this paper we propose a scheme for, and discuss the related issues involved in, such a qualitative angular representation of geometrical objects. Primarily we are concerned here in studying the shape of a polygon that could be encaptured using the *Star-ontology* representation scheme developed in the literature.
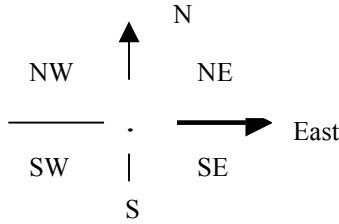

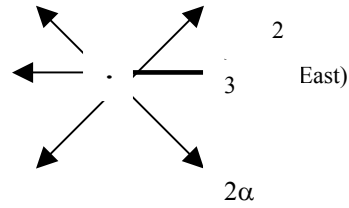Figure 1: Cardinal-direction ontology   Figure 2: A Star-ontology(α)

In "Star-ontology" the point objects are related in a 2D-Euclidean space. The basic relations between a pair of points are relative angular directions between them. For example, in a special case of Star-ontology (Cardinal-directions, [Ligozat, 1998]) the basic relations between a pair of points are {*Equal, East, Northeast, North, Northwest, West, Southwest, South, Southeast*}, where *East, West, North, South* are linear directions, the other four *corner* relations are the four Cartesian quadrants (Figure 1). The angular zoning here is with 90-degrees angle. Star-ontology [Mitra, 2002] generalizes this over any arbitrary angle θ, where $\theta = 360/\alpha$ degrees, for any even integer α. Surprisingly, odd value of α is useless for any representation or reasoning purpose. In the Star-ontology the set of basic relations has $2\alpha + 1$ number of elements. Mitra has designated them [Mitra, 2002] by a set of numbers, {$0 \equiv Equal$, $1 \equiv East$, 2, 3, . . ., 2α}, even numbered relations represent 2D conic-section regions, while the odd numbered relations are the lines radiating like spokes of a wheel from the referred point (Figure 2). *Assertion 1*: In order to represent an n-point polygon it is necessary and sufficient to have n number of binary relations between every ordered pair of consecutive points of the polygon in either clockwise or counter-clockwise sense. *Example*: A quadrilateral is described in α=12 (θ =30-degrees) as {(p1 *2* p2), (p2 *22* p3), (p3 *14* p4), (p4 *8* p1)}, where (p1 *2* p2) means

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point p2 is at zone 2 with respect to point p1. The scheme is obviously inaccurate from a strict quantitative point of view, and a polygon recovered from a given description may be distorted. The maximum distortion of the internal angles in representing polygons with such an approximate description is 2θ (Theorem 1) and the angular deviation of the whole polygon is maximum θ (Theorem 2).

The algorithm for reconstructing the polygon from the approximate description is further influenced by additional constraint of Euclidean geometry that the inner angles must add up to $(n\pi-2\pi)$, for an n-points polygon. This recovery algorithm follows a constraint satisfaction algorithm that first checks if the description is consistent, which is not necessarily true. In general case such a consistency checking problem is NP-hard and is unlikely to have any efficient or polynomial algorithm. However, we have proved [Mitra, 2004] that under a reasonable approximation that the inaccuracy is less than 180-degree, a polynomial algorithm (*path consistency algorithm*, e.g., [Allen, 1983]) will solve the problem.

The proposed qualitative spatial knowledge representation and reasoning scheme is likely to make an automated command, control and communications more efficient.

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